Chapter 10

AC Inductor Design

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Introduction

The design of an ac inductor is quite similar to that of a transformer. If there is no dc flux in the core, the design calculations are straightforward. The apparent power, P_t, of an inductor is the VA of the inductor; that is, the product of the excitation voltage and the current through the inductor.

$$P_t = VA, \quad [\text{watts}] \quad [10\text{-}1]$$

Requirements

The design of the ac inductor requires the calculation of the volt-amp (VA) capability. In some applications the inductance is specified, and in others, the current is specified. If the inductance is specified, then, the current has to be calculated. If the current is specified, then the inductance has to be calculated. A series, ac inductor, L1, being used in a Ferroresonant Voltage Stabilizer is shown in Figure 10-1.
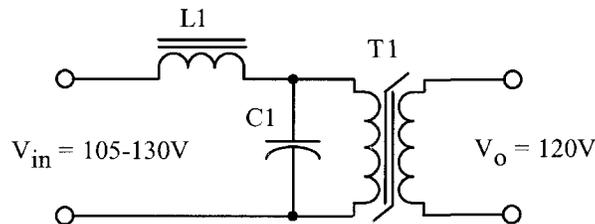

Figure 10-1. Series ac Inductor, L1, as used in a Ferroresonant Voltage Stabilizer.

Relationship of, A_p, to the Inductor Volt-Amp Capability

The volt-amp capability of a core is related to its area product, A_p, by the equation that may be stated as follows:

$$A_p = \frac{VA \left(10^4\right)}{K_f K_u B_{ac} f J}, \quad [\text{cm}^4] \quad [10\text{-}2]$$

Where:

K_f = wave form factor.

K_u = window utilization factor.

B_{ac} = operating flux density, T, tesla

f = operating frequency, Hz

J = current density, amps/cm^2

From the above, it can be seen that factors such as flux density, B_{ac}, the window utilization factor, K_u, (which defines the maximum space occupied by the copper in the window), and the current density, J, all have an influence on the inductor area product, A_p.

Relationship, K_g, to the Inductor Volt-Amp Capability

Although most inductors are designed for a given temperature rise, they can also be designed for a given regulation. The regulation and volt-amp ability of a core is related to two constants:

$$\alpha = \frac{VA}{K_g K_e}, \quad [\%]$$

[10-3]

$$\alpha = \text{Regulation (\%)} \quad [10\text{-}4]$$

The constant, K_g, is determined by the core geometry, which may be related by the following equations:

$$K_g = \frac{W_a A_c^2 K_u}{\text{MLT}}, \quad [\text{cm}^5]$$

[10-5]

The constant, K_e, is determined by the magnetic and electric operating conditions, which may be related by the following equation:

$$K_e = 0.145 K_f^2 f^2 B_m^2 \left(10^{-4}\right)$$

[10-6]

Where:

$$K_f = \text{waveform coefficient}$$

4.0 square wave

4.44 sine wave

From the above, it can be seen that factors such as flux density, frequency of operation, and the waveform coefficient have an influence on the transformer size.

Fundamental Considerations

The design of a linear ac inductor depends upon five related factors:

1. Desired inductance
2. Applied voltage, (across inductor)
3. Frequency
4. Operating Flux density
5. Temperature Rise

With these requirements established, the designer must determine the maximum values for, B_{ac}, which will not produce magnetic saturation, and make trade-offs that will yield the highest inductance for a given volume. The core material selected determines the maximum flux density that can be tolerated for a given design. Magnetic materials and their operating flux levels are given in Chapter 2.

The ac inductor like a transformer, must support the applied voltage, V_{ac}. The number of turns is calculated from Faraday's Law, which states:

$$N = \frac{V_{ac}\left(10^4\right)}{K_f B_{ac} f A_c}, \quad \text{[turns]} \qquad [10\text{-}7]$$

The inductance of an iron-core inductor, with an air gap, may be expressed as:

$$L = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{l_g + \left(\dfrac{\text{MPL}}{\mu_m}\right)}, \quad \text{[henrys]} \qquad [10\text{-}8]$$

Inductance is seen to be inversely dependent on the effective Magnetic Path Length, MPL, which is the sum of the air gap length, l_g, and the ratio of the Magnetic Path Length, MPL, to material permeability, μ_m.

When the core air gap, l_g, is larger compared to the ratio, MPL/μ_m, because of the high material permeability, μ_m, variations in, μ_m, do not substantially affect the total effective Magnetic Path Length, MPL, or the inductance, L. The inductance equation then reduces to:

$$L = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{l_g}, \quad \text{[henrys]} \qquad [10\text{-}9]$$

Rearranging the equation to solve for the gap:

$$l_g = \frac{0.4\pi N^2 A_c \left(10^{-8}\right)}{L}, \quad \text{[cm]} \qquad [10\text{-}10]$$

Fringing Flux

Final determination of the air gap requires consideration of the effect of fringing flux, which is a function of gap dimension, the shape of the pole faces, and the shape, size, and location of the winding, as shown in Figure 10-2 and Figure 10-3. Its net effect is to make the effective air gap less than its physical dimension.

EI Core Configuration
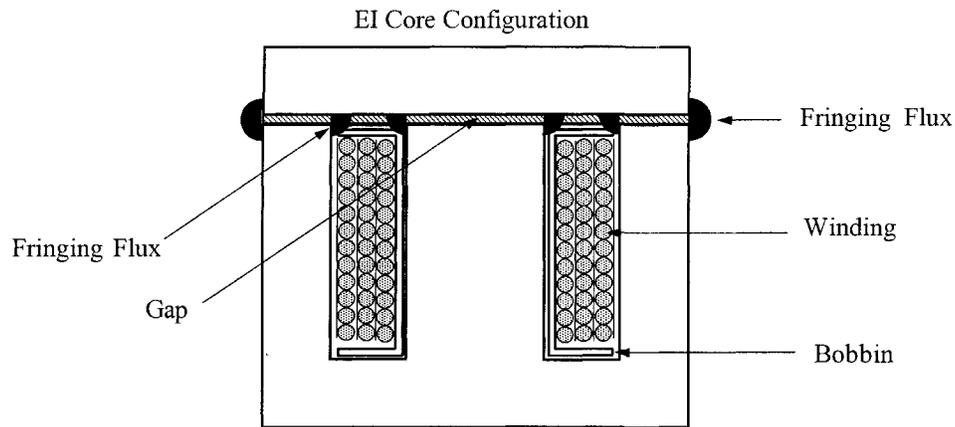
Figure 10-2. Fringing Flux Location on an EI Core Configuration.

EE or Pot Core Configuration
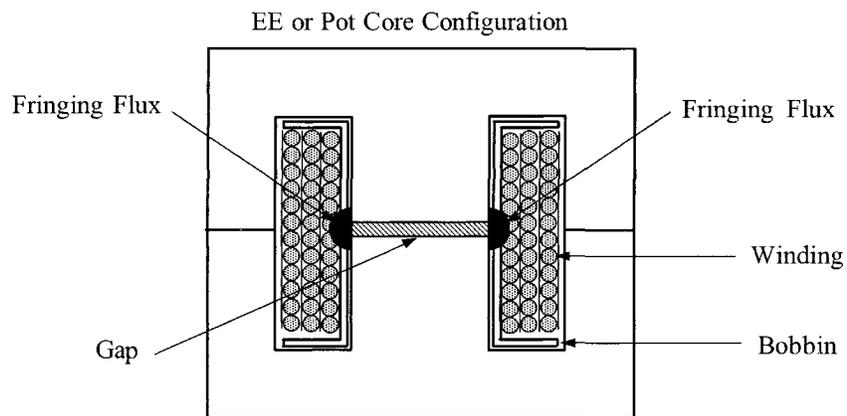
Figure 10-3. Fringing Flux Location on an EE or Pot Core Configuration.

Fringing flux decreases the total reluctance of the magnetic path, and therefore increases the inductance by a factor, F, to a value greater than that calculated from Equation [10-9]. Fringing flux is a larger percentage of the total for larger gaps. The fringing flux factor is:

$$F = \left(1 + \frac{l_g}{\sqrt{A_c}} \ln \left(\frac{2G}{l_g} \right) \right) \qquad [10\text{-}11]$$

Where G is a dimension, (winding length), defined in Chapter 3. Equation [10-11] is valid for cut C cores, laminations and cut ferrite cores.

The inductance, L, computed in Equation [10-9] does not include the effect of fringing flux. The value of inductance, L', in Equation [10-12] does correct for fringing flux:

$$L' = \frac{0.4\pi N^2 A_c F\left(10^{-8}\right)}{l_g}, \quad [\text{henrys}] \qquad [10\text{-}12]$$

Now that the fringing flux, F, has been calculated, it is necessary to recalculate the number of turns using the fringing flux, Factor F.

$$N_{(new)} = \sqrt{\frac{L l_g}{0.4\pi A_c F\left(10^{-8}\right)}}, \quad [\text{turns}] \qquad [10\text{-}13]$$

After the new turns, $N_{(new)}$, have been calculated, then, use Equation [10-13] with the new turns, $N_{(new)}$, and solve for B_{ac}. This check will provide the operating flux density, in order to calculate the core loss, P_{fe}, and will also provide a check on core saturation margin.

$$B_{ac} = \frac{V_{ac}\left(10^4\right)}{K_f N_{(new)} f A_c}, \quad [\text{tesla}] \qquad [10\text{-}14]$$

The losses in an ac inductor are made up of three components:
1. Copper loss, P_{cu}
2. Iron loss, P_{fe}
3. Gap loss, P_g

The copper loss, P_{cu}, is $I^2 R$ and is straightforward, if the skin effect is minimal. The iron loss, P_{fe}, is calculated from core manufacturers' data. Gap loss, P_g, is independent of core material strip thickness and permeability. Maximum efficiency is reached in an inductor, as in a transformer, when the copper loss, P_{cu}, and the iron loss, P_{fe}, are equal, but only when the gap of the core is zero. The gap loss does not occur in the air gap, itself, but is caused by magnetic flux, fringing around the gap, and reentering the core in a direction of high loss. As the air gap increases, the flux across the gap fringes more and more, and some of the fringing flux strikes the core, perpendicular to the laminations, and sets up eddy currents which cause additional losses called gap loss, P_g. Also distribution of the fringing flux is affected by other aspects of the core geometry, the proximity of the coils turns to the core, and whether there are turns on both legs. (See Table 10-1). Accurate prediction of the gap loss depends on the amount of fringing flux. (See the Reference at the end of this Chapter)

$$P_g = K_i E l_g f B_{ac}^2, \quad [\text{watt}] \quad [10\text{-}15]$$

Where, E (as defined in Chapter 3) is the strip or tongue width, in cm.

Table 10-1. Gap Loss Coefficient

Configuration	K_i
Two-coil C core	0.0388
Single-coil C core	0.0775
Lamination	0.1550

When designing inductors where there is a choice of cores, always pick the core with the smallest ratio:

$$\frac{W_a}{A_c} = \text{[smallest ratio]} \quad \text{[10-16]}$$

Comparing two cores with identical area products, A_p, for the same design specification, the core with a minimum of window area will generate a minimum of fringing flux. If there is a design change and it requires the use of the next larger core, it would be far more beneficial to double up on the core being used, than to pick a larger core, as shown in Figure 10-4.
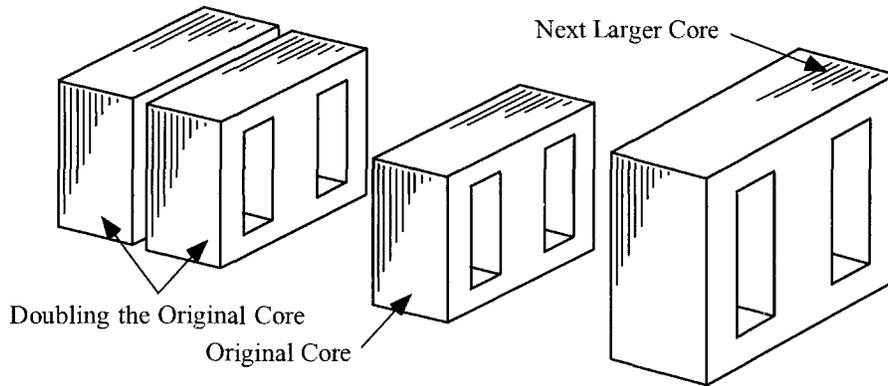

Figure 10-4. Comparing Core Configurations.

For example, if the next larger core was selected, normally all of the core proportions will increase. This means, the window, W_a, and the iron cross-section, A_c, would have both increased. (A larger core should not be used as the fringing flux would also increase.) If you want to keep the fringing flux to a minimum, then double up on the original core. Therefore the iron area, A_c, would double, but the window area, W_a, will remain the same. This will reduce the, W_a/A_c, ratio, as shown in Equation 10-16. With an increase in iron cross-section, A_c, the turns would have to decrease for the same window area, W_a. With a decrease in turns, the gap would also decrease, resulting in less fringing flux.

When designing a transformer, the engineer will push the flux density as far he can without saturating the core. That cannot be done with an ac inductor because you must leave a margin for the fringing flux factor. One of the biggest problems in designing ac inductors is keeping the gap to a minimum. This problem becomes acute when designing high frequency ac inductors. The problem in designing high frequency inductors is the required turns to support the applied voltage, then gapping to provide the proper inductance. This problem is minimized when using powder cores if the right permeability can be found.

AC Inductor Design Example

Step No. 1 Design a linear ac inductor with the following specifications.

1. Applied voltage, V_L .... = 120 volts

2. Line current, I_L .... = 1.0 amps

3. Line frequency .... = 60 hertz.

4. Current density, J .... = 300 amp/cm^2

5. Efficiency goal, $\eta(100)$ .... = 90%

6. Magnetic material .... = Silicon

7. Magnetic material permeability, μ_m .... = 1500

8. Flux density, B_{ac} .... = 1.4 tesla

9. Window utilization, K_u .... = 0.4

10. Waveform factor, K_f .... = 4.44

11. Temperature rise goal, T_r .... = 50°C

Step No. 2 Calculate the apparent power, P_t or VA of the inductor, L.

$$VA = V_L I_L, \quad [\text{watts}]$$
$$VA = (120)(1.0), \quad [\text{watts}]$$
$$VA = 120, \quad [\text{watts}]$$

Step No. 3 Calculate the area product, A_p.

$$A_p = \frac{VA(10^4)}{K_f K_u f B_{ac} J}, \quad [\text{cm}^4]$$

$$A_p = \frac{(120)(10^4)}{(4.44)(0.4)(60)(1.4)(300)}, \quad [\text{cm}^4]$$

$$A_p = 26.8, \quad [\text{cm}^4]$$

Step No. 4 Select an EI lamination from Chapter 3. The closest lamination to the calculated area product, A_p, is the EI-100.

1. Core Number .... = EI-100

2. Magnetic Path Length, MPL .... = 15.2 cm

3. Core Weight, W_{tfe} .... = 676 grams

4. Mean Length Turn, MLT .... = 14.8 cm

5. Iron Area, A_c .... = 6.13 cm^2

6. Window Area, W_a .... = 4.84 cm^2

7. Area Product, A_p .... = 29.7 cm^4

8. Core Geometry, K_g .... $= 4.93$ cm^5

9. Surface Area, A_t .... $= 213$ cm^2

10. Winding length, G .... $= 3.81$ cm

11. Lamination tongue, E .... $= 2.54$ cm

Step No. 5 Calculate the number of inductor turns, N_L.

$$N_L = \frac{V_L(10^4)}{K_f B_{ac} f A_c}, \quad [\text{turns}]$$

$$N_L = \frac{(120)(10^4)}{(4.44)(1.4)(60)(6.13)}, \quad [\text{turns}]$$

$$N_L = 525, \quad [\text{turns}]$$

Step No. 6 Calculate the inductive reactance, X_L.

$$X_L = \frac{V_L}{I_L}, \quad [\text{ohms}]$$

$$X_L = \frac{(120)}{(1.0)}, \quad [\text{ohms}]$$

$$X_L = 120, \quad [\text{ohms}]$$

Step No. 7 Calculate the required inductance, L.

$$L = \frac{X_L}{2\pi f}, \quad [\text{henrys}]$$

$$L = \frac{120}{2(3.14)(60)}, \quad [\text{henrys}]$$

$$L = 0.318, \quad [\text{henrys}]$$

Step No. 8 Calculate the required gap, L_g.

$$l_g = \left(\frac{0.4\pi N_L^2 A_c(10^{-8})}{L}\right) - \left(\frac{MPL}{\mu_m}\right), \quad [\text{cm}]$$

$$l_g = \left(\frac{(1.26)(525)^2(6.13)(10^{-8})}{0.318}\right) - \left(\frac{15.2}{1500}\right), \quad [\text{cm}]$$

$$l_g = 0.0568, \quad [\text{cm}] \text{ or } l_g = 22.4, \quad [\text{mils}]: \text{This would be in 10 mils each leg.}$$

Step No. 9 Calculate the fringing flux, F.

$$F = \left(1 + \frac{l_g}{\sqrt{A_c}} \ln \frac{2(G)}{l_g}\right)$$

$$F = \left(1 + \frac{0.0568}{\sqrt{6.13}} \ln \frac{2(3.81)}{0.0568}\right)$$

$$F = 1.112$$

Step No. 10 Using the fringing flux, recalculate the series inductor turns, $N_{L(new)}$.

$$N_{L(new)} = \sqrt{\frac{l_g L}{0.4\pi A_c F(10^{-8})}}, \quad [\text{turns}]$$

$$N_{L(new)} = \sqrt{\frac{(0.0568)(0.318)}{(1.26)(6.13)(1.112)(10^{-8})}}, \quad [\text{turns}]$$

$$N_{L(new)} = 459, \quad [\text{turns}]$$

Step No. 11 Using the new turns, recalculate the flux density, B_{ac}.

$$B_{ac} = \frac{V_L(10^4)}{K_f N_{L(new)} A_c f}, \quad [\text{tesla}]$$

$$B_{ac} = \frac{(120)(10^4)}{(4.44)(459)(6.13)(60)}, \quad [\text{tesla}]$$

$$B_{ac} = 1.6, \quad [\text{tesla}]$$

Step No. 12 Calculate the inductor bare wire area, $A_{wL(B)}$.

$$A_{wL(B)} = \frac{I_L}{J}, \quad [\text{cm}^2]$$

$$A_{wL(B)} = \frac{(1.0)}{(300)}, \quad [\text{cm}^2]$$

$$A_{wL(B)} = 0.00333, \quad [\text{cm}^2]$$

Step No. 13 Select a wire from the Wire Table in Chapter 4.

$$AWG = \#22$$

$$A_{w(B)} = 0.00324, \quad [\text{cm}^2]$$

$$\left(\frac{\mu\Omega}{\text{cm}}\right) = 531, \quad [\text{micro-ohm/cm}]$$

Step No. 14 Calculate the inductor winding resistance, R_L. Use the MLT from the core data found in Step 4 and the micro-ohm per centimeter found in Step 13.

$$R_L = (\text{MLT})(N_s)\left(\frac{\mu\Omega}{\text{cm}}\right)(10^{-6}), \quad [\text{ohms}]$$

$$R_L = (14.8)(459)(531)(10^{-6}), \quad [\text{ohms}]$$

$$R_L = 3.61, \quad [\text{ohms}]$$

Step No. 15 Calculate the inductor winding copper loss, P_L.

$$P_L = (I_L)^2 R_L, \quad [\text{watts}]$$

$$P_L = (1.0)^2 (3.61), \quad [\text{watts}]$$

$$P_L = 3.61, \quad [\text{watts}]$$

Step No. 16 Calculate the watts-per-kilograms, W/K, for the appropriate core material. See Chapter 2.

$$W/K = 0.000557 f^{(1.68)} B_s^{(1.86)}, \quad [\text{watts-per-kilogram}]$$

$$W/K = 0.000557 (60)^{(1.68)} (1.6)^{(1.86)}, \quad [\text{watts-per-kilogram}]$$

$$W/K = 1.30, \quad [\text{watts-per-kilogram}]$$

Step No. 17 Calculate the core loss in watts, P_{fe}.

$$P_{fe} = (W/K)W_{tfe}, \quad [\text{watts}]$$

$$P_{fe} = (1.30)(0.676), \quad [\text{watts}]$$

$$P_{fe} = 0.878, \quad [\text{watts}]$$

Step No. 18 Calculate the gap loss, P_g.

$$P_g = K_i E l_g f B_{ac}^2, \quad [\text{watts}]$$

$$P_g = (0.155)(2.54)(0.0568)(60)(1.6)^2, \quad [\text{watts}]$$

$$P_g = 3.43, \quad [\text{watts}]$$

Step No. 19 Calculate the total inductor losses, P_Σ.

$$P_\Sigma = P_{cu} + P_{fe} + P_g, \quad [\text{watts}]$$

$$P_\Sigma = (3.61) + (0.878) + (3.43), \quad [\text{watts}]$$

$$P_\Sigma = 7.92, \quad [\text{watts}]$$

Step No. 20 Calculate the inductor surface area watt density, ψ.

$$\psi = \frac{P_\Sigma}{A_t}, \quad [\text{watts-per-cm}^2]$$

$$\psi = \frac{(7.92)}{(213)}, \quad [\text{watts-per-cm}^2]$$

$$\psi = 0.0372, \quad [\text{watts-per-cm}^2]$$

Step No. 21 Calculate the temperature rise, T_r.

$$T_r = 450(\psi)^{(0.826)}, \quad [^\circ C]$$

$$T_r = 450(0.0372)^{(0.826)}, \quad [^\circ C]$$

$$T_r = 29.7, \quad [^\circ C]$$

Step No. 22 Calculate the window utilization, K_u.

$$K_u = \frac{N_{L(new)} A_{w(B)\#22}}{W_a}$$

$$K_u = \frac{((459)(0.00324))}{(4.84)}$$

$$K_u = 0.307$$

Reference

1. Ruben, L., and Stephens, D. Gap Loss in Current-Limiting Transformer. Electromechanical Design, April 1973, pp. 24-126.